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Natural convection heat transfer in enclosures with multiple vertical partitions

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Abstract—Laminar natural convection in rectangular enclosures divided by multiple vertical partitions is studied experimentally and by numerical calculation. In the boundary layer regime, the partition temperature approximately increases linearly in the vertical direction. The boundary layer solution predicting the heat transfer rate is derived on the basis of the numerical results. It is shown that the Nusselt number is inversely proportional to $(1+N)$ where N is the number of partitions. This is also confirmed by the experiments.

1. INTRODUCTION

NATURAL convection in enclosures is a topic of considerable engineering interest. Applications range from thermal design of buildings, to cryogenic storage, solar collector design, nuclear reactor design, and others. Several excellent reviews of the literature [1, 2] have been published.

The problem of primary interest in the literature is that of an enclosure with no partitions. However, in practical cases, vertical partitions are inserted into the enclosure to reduce heat losses by natural convection and thermal radiation. There have been several studies on the effect of a single vertical partition in suppressing natural convection, including the case of a porous medium [3–10].

In particular, the present authors [5, 9] proposed a boundary layer solution for this system and confirmed its validity by experiments. It is found that the partition has the effect of reducing the heat transfer rate by about 55% at high Rayleigh numbers. Thus it is expected that the suppression of natural convection becomes more significant when the multiple vertical partitions are inserted into an enclosure.

Natural convection in enclosures with multiple vertical partitions is relatively unknown, in fact we are aware of only two fundamental studies [11, 12]. Anderson and Bejan [11] measured the heat transfer rates through double partitions which are inserted in the middle of an enclosure and indicated that the heat transfer rate for double partitions is 20% smaller than that for a single partition. Jones [12] demonstrated numerical results from the IOTA2 code for laminar buoyancy driven flows in rectangular enclosures. One of the results was presented for five partitions, and he found that the effect of dividing the enclosure into six cells reduces the heat transfer rate by, approximately, a factor of 6.

From the above literature, it appears that there is a great lack of generalized information on multiple vertical partitions. In this paper, laminar natural convection in each cell constructed by multiple vertical partitions equally spaced in rectangular enclosures as shown in Fig. 1 was studied experimentally and by numerical computation.

2. EXPERIMENTAL APPARATUS AND PROCEDURE

Figure 2 shows a schematic diagram of the experimental apparatus. The main parts of the apparatus are the cooling part, the test section part and the heating part. The test section was a rectangular enclosure which consisted of a lucite frame (20 mm thick)

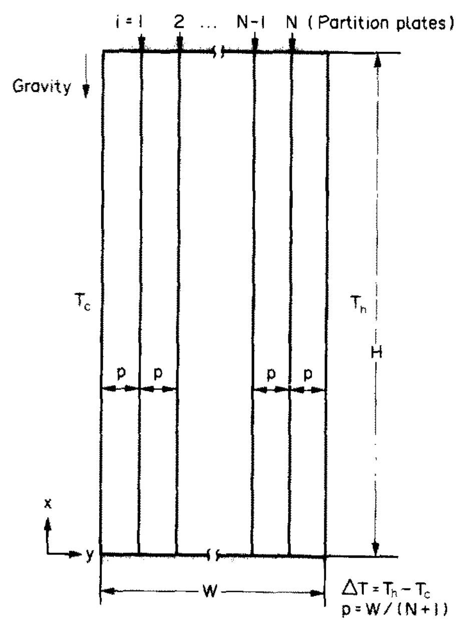

FIG. 1. Schematic diagram of an enclosure with partitions.

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NOMENCLATURE

- B temperature difference between the partition and the core region
- G temperature gradient in the vertical direction
- g gravitational acceleration
- H height of the enclosure
- h heat transfer coefficient, $Q/\Delta T$
- Nu Nusselt number, hW/λ
- Nu_H Nusselt number based on H, hH/λ
- Nu_x local Nusselt number
- L depth of the enclosure
- N number of partitions
- Pr Prandtl number
- Ra Rayleigh number, $g\beta\Delta TW^3/(\nu\alpha)$
- Ra_H Rayleigh number based on H, $g\beta\Delta TH^3/(\nu\alpha)$
- p pitch between the partitions
- Q heat flux through the enclosure
- T temperature
- T_c temperature at the cold wall
- T_h temperature at the hot wall
- ΔT temperature difference, $T_h - T_c$
- U dimensionless vertical velocity, uW/α
- u vertical velocity
- V dimensionless horizontal velocity, vW/α
- v horizontal velocity
- X dimensionless vertical coordinate, x/W
- x vertical coordinate
- Y dimensionless horizontal coordinate, y/W
- y horizontal coordinate.

Greek symbols

- α thermal diffusivity
- β volumetric expansion coefficient
- η reducing rate of heat transfer defined by equation (17)
- θ dimensionless temperature, $(T-T_c)/(T_h-T_c)$
- λ thermal conductivity
- ν kinematic viscosity
- Ψ dimensionless stream function, ψ/α
- ψ stream function
- Ω dimensionless vorticity, $\omega W^2/\alpha$
- ω vorticity.

placed between two copper plates (12 mm thick) corresponding to the hot and cold walls. The following two kinds of enclosures were used in this study; the height and length of the enclosure were fixed ($H =$ 300 mm and $L = 200$ mm) and the width was variable ($W = 30$ and 75 mm). The heating part consisted of the main heaters and the guard heaters mounted on the rear side of the main heaters across a bakelite plate. These heaters were divided into four parts to maintain a uniform temperature distribution on the hot wall. A cooling chamber was attached to the rear side of the cold wall, and the wall temperature was maintained uniform by introducing a sufficiently large amount of temperature-controlled brine from a

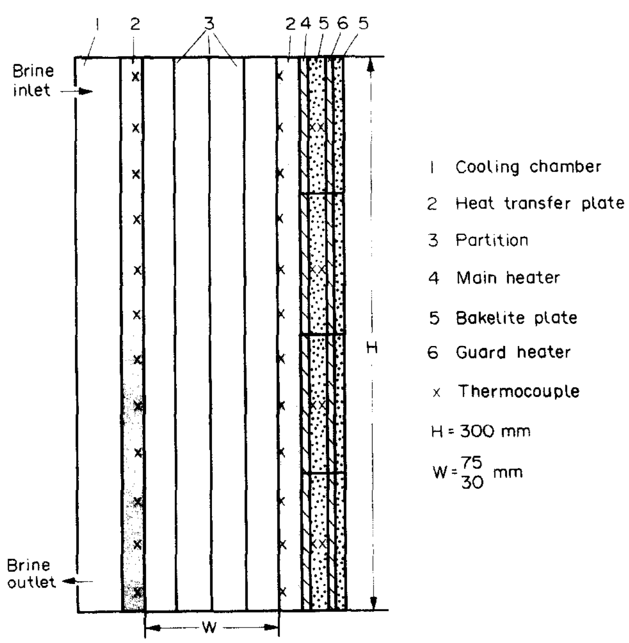

FIG. 2. Experimental apparatus.

refrigerator. The temperatures were measured using 100 μm diameter copper–constantan thermocouples fixed at the positions shown in Fig. 2. In order to minimize the heat loss, the experimental apparatus was covered with polystyrene foam insulating material 60 mm thick and in addition the apparatus was located in a temperature-controlled room. The partition was made of a thin copper plate, 100 μm thick, and the number of partitions was varied from 1 to 4. The working fluid was water. The experiments were carried out in the range $10^6 < Ra < 10^8$.

3. EXPERIMENTAL RESULTS

The heat transfer rate across the enclosure was measured electrically, by monitoring the power dissipated in the main heaters (Fig. 2). The heat transfer measurements for $H/W = 4$ are presented in Fig. 3, where the average Nusselt number and Rayleigh number are defined as follows:

$$Nu = \frac{QW}{\lambda \Delta T} \tag{1}$$

$$Ra = \frac{g\beta \Delta T W^3}{\nu\alpha}. \tag{2}$$

The physical properties appearing in the above definitions have been evaluated at the end-to-end average temperature $0.5(T_h + T_c)$.

The Nusselt number for no partitions is in good agreement with the correlation by Churchill [2], which is a generalization of the laminar boundary layer solution of Bejan, including the effect of Prandtl number.

In the case of partition, the Nusselt number decreases drastically on increasing the number of partitions N, but the introduction of the partitions does not produce a proportional reduction in heat transfer. In the range of Rayleigh numbers considered here, the Rayleigh number dependence is almost identical for all cases, i.e. Nu is approximately proportional to $Ra^{0.25}$. It appears that the dominant mode of heat transfer in this experimental range is a boundary layer type.

Duxbury [3] and Nakamura *et al.* [4] indicated that an isothermal partition model which assumes the partition to be isothermal can be used to estimate approximately the heat transfer rate across the enclosure with a single partition. We applied the isothermal partition model to the case of multiple partitions and estimated the Nusselt numbers. Fine lines also shown in this figure represent the heat transfer correlations obtained by the isothermal partition model, indicating that the Nusselt number is proportional to $(N+1)^{-1.25}$. This model seems to predict the heat transfer rate for $N = 1$, but tends to underpredict the heat transfer rate with an increase of N. The reason for this is that in fact the partition is not isothermal but its temperature increases in the vertical direction due to thermal stratification of the fluid within both cells divided by the partition. We therefore examined the detailed flow and temperature fields in each cell by means of numerical computation.

4. NUMERICAL COMPUTATION

The flow in the enclosure is assumed to be two-dimensional with velocity components u and v along the x- and y-coordinates, respectively (Fig. 1). Also the fluid is assumed to conform to the Boussinesq approximation whereby the density varies linearly with temperature.

In terms of dimensionless variables of stream function, vorticity and temperature, the flow and temperature are governed by the following set of conservation equations:

$$U\frac{\partial \Omega}{\partial X} + V\frac{\partial \Omega}{\partial Y} = Pr\nabla^2\Omega - Ra\,Pr\frac{\partial \theta}{\partial Y} \tag{3}$$

$$\nabla^2\Psi = -\Omega \tag{4}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \nabla^2\theta. \tag{5}$$

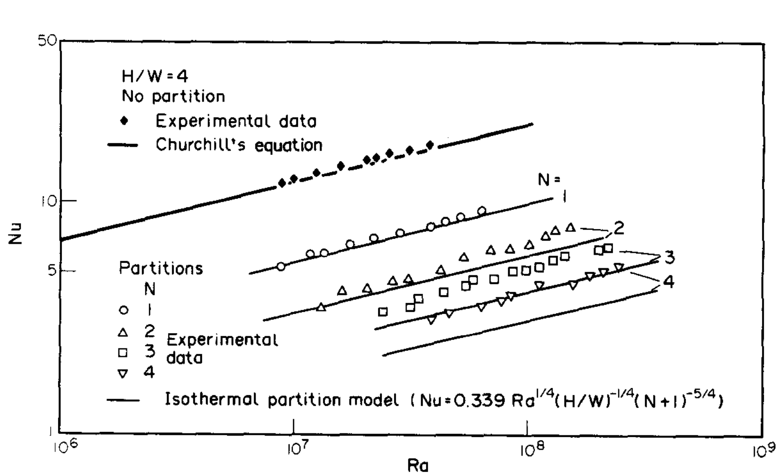

FIG. 3. Experimental Nusselt number vs Rayleigh number for $H/W = 4$.

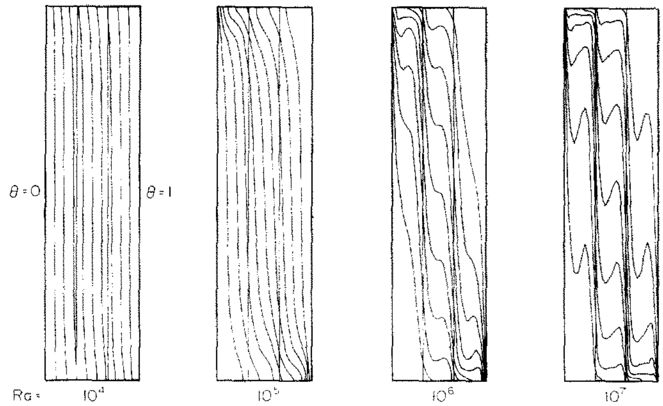

FIG. 4. Isotherms for $H/W = 4$ and $N = 2$.

The boundary conditions at four walls of the enclosure are given as

at $Y = 0$, $X = 0-H/W$:

$$\Psi = 0, \quad \Omega = -\nabla^2\Psi, \quad \theta = 0 \quad (6)$$

at $Y = 1$, $X = 0-H/W$:

$$\Psi = 0, \quad \Omega = -\nabla^2\Psi, \quad \theta = 1 \quad (7)$$

at $Y = 0-1$, $X = H/W$:

$$\Psi = 0, \quad \Omega = -\nabla^2\Psi, \quad \partial\theta/\partial X = 0 \quad (8)$$

at $Y = 0-1$, $X = 0$:

$$\Psi = 0, \quad \Omega = -\nabla^2\Psi, \quad \partial\theta/\partial X = 0. \quad (9)$$

At the partitions, the condition of continuity of temperature and temperature gradient are imposed. Of course the velocity components are zero. These conditions are given in terms of variables as

$$\Psi_{p^\pm} = 0, \quad \Omega_{p^+} = -\nabla^2\Psi_{p^+}, \quad \theta_{p^-} = \theta_{p^+},$$
$$\partial\theta/\partial Y_{p^-} = \partial\theta/\partial Y_{p^+}. \quad (10)$$

The solution of these equations is dependent on the following parameters: *Pr*, *Ra*, H/W and *N*. The aspect ratio and Prandtl number were held constant ($H/W = 4$ and $Pr = 6$ which correspond to the above experiments). Other parameters were varied in the range $10^4 < Ra < 10^7$, and $N = 2$ and 3.

Because of the conditions of continuity of temperature and heat flux across the partitions, the implicit calculation needed for the temperature field. That is, in the finite difference method the energy equation is initially solved for each cell by assuming the temperature distribution along the partitions and then the calculation is repeated until the solution converged [6]. This procedure becomes complex as the number of partitions increases. On the other hand, in the Galerkin finite element method the energy equation can be solved for the enclosure as a whole, since a weighted residual equation used in the calculation of temperature automatically satisfies these boundary conditions. Thus equations (3)–(10) were solved using the Galerkin finite element method.

The solution technique is described in the literature well [13] and has been widely used for natural convection problems [14, 15]. The results for $N = 2$ and 3 are obtained by a 50×39 and a 50×40 mesh, respectively.

5. RESULTS OF NUMERICAL COMPUTATION

Figure 4 shows isotherms at $N = 2$ for various Rayleigh numbers. At $Ra = 10^4$, isotherms are almost parallel to the side walls of the enclosure, indicating that most of the heat transfer is by heat conduction. As the Rayleigh number increases, the isotherms undulate remarkably and thus the effect of convection is pronounced. At $Ra = 10^7$, the density of isotherms is more severe near the side walls and the partitions, but diminishes in the middle part of each cell, indicating formation of a thermal boundary layer. Hereafter we focus on the temperature field in the boundary layer regime.

Figure 5 shows the horizontal temperature profiles for $Ra = 10^7$ at different levels. The existence of the thermal boundary layers along the partitions is indicated clearly and the middle part of each cell is regarded as the core region. Figure 6 shows the par-

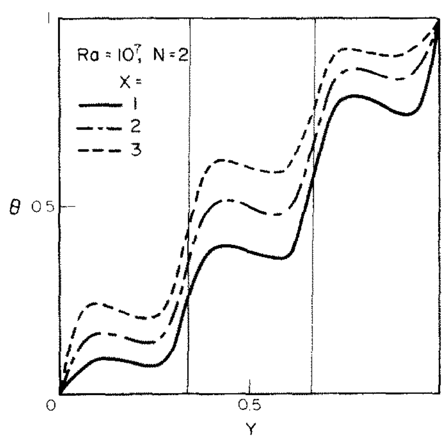

FIG. 5. Temperature profiles at horizontal section for $H/W = 4$ and $N = 2$ at $Ra = 10^7$.

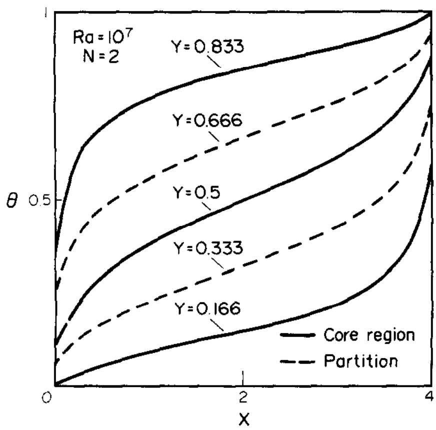

FIG. 6. Temperature profiles at vertical section for $H/W = 4$ and $N = 2$ at $Ra = 10^7$.

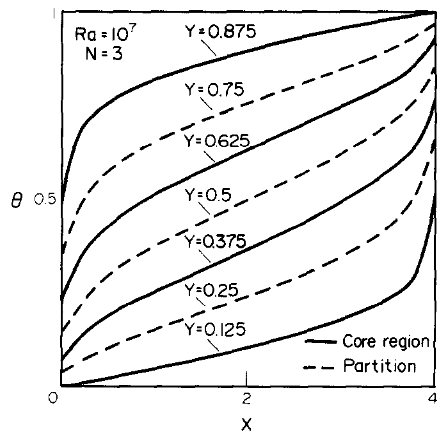

FIG. 8. Temperature profiles at vertical section for $H/W = 4$ and $N = 3$ at $Ra = 10^7$.

tition and core temperature profiles in the vertical direction. The solid lines denote the core temperatures ($Y = 0.166$, 0.5 and 0.833) and the dotted lines are the partition temperatures ($Y = 0.333$ and 0.666). The temperature profile of each partition exhibits a linear variation in the vertical direction except near the lower and upper walls of the enclosure, indicating that the isothermal partition model mentioned in Section 3 is not appropriate. The core temperature on both sides of each partition does not increase at the same rate as the partition temperature, unlike the case of $N = 1$ previously reported [5]. Thus the local heat flux through each partition is not uniform but varies in the vertical direction as shown in Fig. 7.

Figure 8 shows the results for $N = 3$. The temperature variations of the outer partition ($Y = 0.25$ or 0.75) and core regions on both sides of its partition are similar to those for $N = 2$. However, the temperature variations of the central partition ($Y = 0.5$) and core regions on both sides of its partition show the same behavior as those for $N = 1$. That is, the core temperature increases linearly at nearly the same rate as the partition temperature. Therefore, as shown in Fig. 9, the local heat flux varies in the vertical direction at the outer partition, like the case of $N = 2$, but is almost uniform at the central position, like the case of $N = 1$.

It is deduced from the above results that the local heat flux is uniform at the central partition ($Y = 0.5$) for an odd number of partitions. We try to estimate the heat transfer rate through the central partition from a simple boundary layer model previously applied to the case of $N = 1$ [5]. This boundary layer model includes the following assumptions. The partition temperature varies linearly in the vertical direction, a constant temperature gradient being positive. The temperature gradient at the core region on both sides of the partition is the same as that at the partition, but the temperature is different. These temperature profiles are represented as

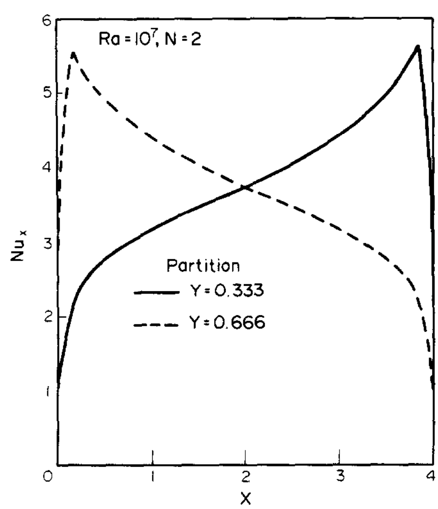

FIG. 7. Local Nusselt number profiles for $H/W = 4$ and $N = 2$ at $Ra = 10^7$.

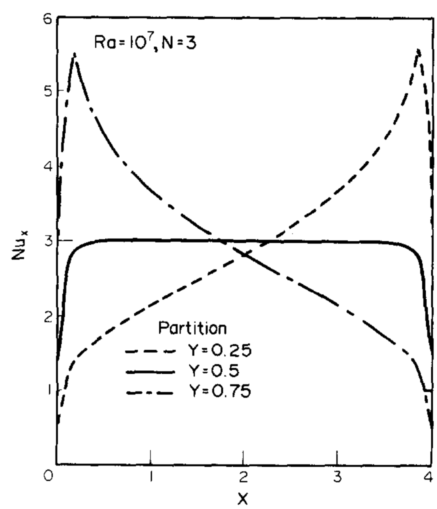

FIG. 9. Local Nusselt number profiles for $H/W = 4$ and $N = 3$ at $Ra = 10^7$.

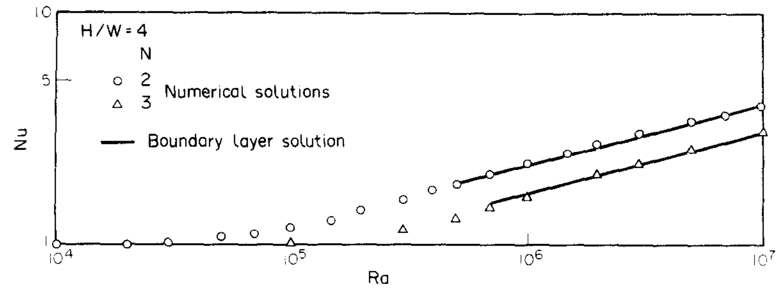

FIG. 10. Comparison of numerical and boundary layer solutions in Nusselt number for $N = 2$ and 3.

$$\text{partition plate}: T = B+Gx \tag{11}$$

$$\text{core region}: T = Gx \tag{12}$$

where B is the temperature difference between the partition and the core region, and G the temperature gradient. These values are determined on the basis of the computational results for the cases of $N = 1$ and 3, and therefore the following simple relations are obtained:

$$B = \Delta T/2(N+1) \tag{13}$$

$$(N = 1, 3, \ldots)$$

$$G = \Delta T/2H. \tag{14}$$

The Nusselt number is given by

$$Nu = QW/(\lambda\Delta T) = BW/(l\Delta T) \tag{15}$$

where l represents a scale of thermal boundary layer thickness ($= (4\alpha\nu/\beta gG)^{1/4}$). Derivation of equation (15) has been described in the previous study [5].

Thus the Nusselt number correlation is represented as

$$Nu = 0.297Ra^{1/4}(H/W)^{-1/4}(N+1)^{-1}. \tag{16}$$

It is noted that the Nusselt number is proportional to $(N+1)^{-1}$ instead of $(N+1)^{-1.25}$ given by the isothermal partition model mentioned in Section 3. This correlation predicts the heat transfer rate well not only for an odd number of partitions but also for an even number as shown in Fig. 10. The agreement for an even number of partitions is fortunate because the physical situation is not valid in this boundary layer model.

Furthermore, in order to confirm the generality of this correlation we compare it with the experimental data in Section 3. The results are shown in Fig. 11. The referential length in the Nusselt number and the Rayleigh number is the height of the enclosure, instead of the width because the effect of aspect ratio does not appear in the correlation using this definition. All of the experimental data for $H/W = 4$ and 10 agree well with the boundary layer solution indicated by the solid line, in particular for an even number of partitions. Thus it is evident that the heat transfer correlation of equation (16) is very useful for the prediction of heat transfer rate in the enclosure with multiple vertical partitions. Figure 12 shows a comparison of the present boundary layer solution for $N = 1$ and numerical solutions previously reported for several values of H/W and Pr [5–8]. This result deduces that the present correlation is applicable to air as well as water.

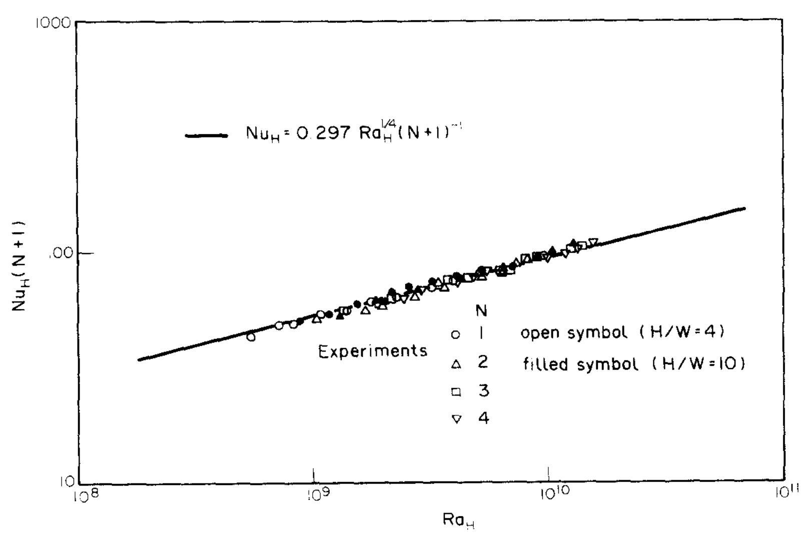

FIG. 11. Comparison of experimental data and boundary layer solution in Nusselt number.

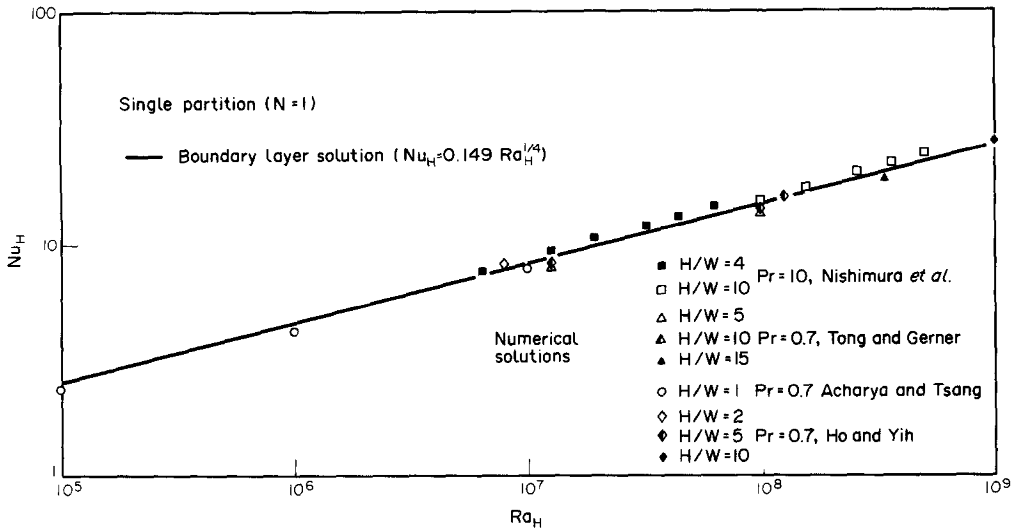

FIG. 12. Comparison of numerical and boundary layer solutions for $N = 1$.

Returning to the engineering application which motivated this fundamental study, we now assess the thermal insulation capability of the multiple vertical partitions. The reducing rate of heat transfer η is defined as

$$\eta = 1 - Nu_p/Nu_0 \tag{17}$$

where Nu_p is the Nusselt number with the partitions of equation (16), and Nu_0 is the Nusselt number without the partitions which is obtained from the correlation of Churchill [2]. Figure 13 shows the relation between the heat transfer reduction and the number of partitions. The value of η increases with the number of partitions, but the rate of increase gradually decreases. It is noted from an engineering standpoint that a useful number of partitions is 2–5, having the effect of reducing the heat transfer rate by 70–90%.

6. CONCLUSIONS

Natural convection heat transfer in rectangular enclosures with multiple vertical partitions was investigated experimentally and by numerical computation. The enclosure was bounded by isothermal vertical walls at different temperatures and adiabatic horizontal walls. The partitions were equally spaced in the enclosure and the thickness of partitions was neglected.

(1) In the boundary layer regime, the partition and core temperatures approximately increase linearly in the vertical direction except near the upper and lower walls of the enclosure. The local heat flux through the partitions varies in the vertical direction in most cases, but it is uniform at the central partition for an odd number of partitions.

(2) The boundary layer solution is derived on the basis of the computational results and the heat transfer correlation is presented as

$$Nu = 0.297 Ra^{1/4} (H/W)^{-1/4} (N+1)^{-1}.$$

This correlation predicts well the experimental data performed additionally and the numerical results by other investigators.

(3) From an engineering standpoint, a useful number of partitions is 2–5, having the effect of reducing the heat transfer rate by 70–90%.

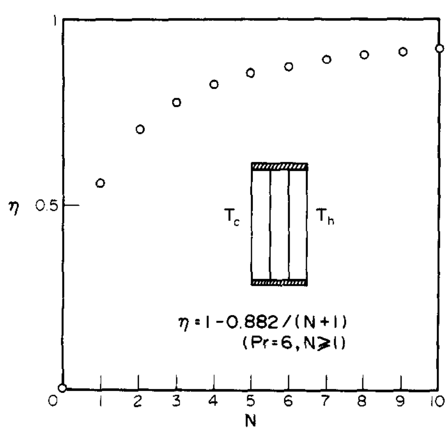

FIG. 13. Heat transfer reduction due to the presence of partitions.

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CONVECTION THERMIQUE NATURELLE DANS DES ENCEINTES AYANT DES PARTITIONS VERTICALES

Résumé—On étudie expérimentalement et numériquement la convection naturelle laminaire dans des cavités rectangulaires divisées par des partitions verticales multiples. Dans le régime de couche limite, la température de partition augmente à peu près linairement dans la direction verticale. La solution de couche limite qui prédit le flux thermique transféré est dérivée à partir des résultats numériques. On montre que le nombre de Nusselt est inversement proportionnel à $(1+N)$, si N est le nombre de partitions. Cela est confirmé par les expériences.

WÄRMEÜBERGANG AUFGRUND NATÜRLICHER KONVEKTION IN BEHÄLTERN MIT MEHREREN VERTIKALEN TRENNWÄNDEN

Zusammenfassung—Die laminare natürliche Konvektion in rechtwinkligen Behältern mit mehreren senkrechten Trennwänden wurde experimentell und numerisch untersucht. Im Grenzschichtbereich steigt die Trennwandtemperatur in vertikaler Richtung annähernd linear an. Die Lösung des Grenzschichtproblems, aus der man den Wärmeübergangskoeffizienten erhält, wurde von den numerischen Ergebnissen abgeleitet. Es wurde gezeigt, daß die Nusselt-Zahl umgekehrt proportional zu $(1+N)$ ist (N ist die Anzahl der Trennwände). Dies wird auch durch die Versuche bestätigt.

ЕСТЕСТВЕННОКОНВЕКТИВНЫЙ ТЕПЛОПЕРЕНОС В ПОЛОСТЯХ С ВЕРТИКАЛЬНЫМИ ПЕРЕГОРОДКАМИ

Аннотация—Экспериментально и численно исследуется ламинарная естественная конвекция в прямоугольных полостях, разделенных рядом вертикальных перегородок. В режиме пограничного слоя температура перегородок возрастает линейно в вертикальном направлении. На основе численных расчетов получено решение системы уравнений пограничного слоя, которое определяет коэффициент теплопередачи. Показано, что число Нуссельта обратно пропорционально величине $(1 + N)$, где N—число перегородок. Эксперименты подтверждают этот результат.